

Top quark associated production of topcolor pions at hadron colliders

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Abstract

We investigate the associated production of a neutral physical pion with top quarks in the context of topcolor assisted technicolor. We find that single-top associated production does not yield viable rates at either the Tevatron or LHC. $t\bar{t}$ -associated production at the Tevatron is suppressed relative to Standard Model $t\bar{t}H$, but at the LHC is strongly enhanced and would allow for easy observation of the main decay channels to bottom quarks, and possible observation of the decay to gluons.

I. INTRODUCTION

Hadron colliders are machines extremely well-suited to study the forefront problem of electroweak symmetry breaking (EWSB) and fermion mass generation. Fermilab's Tevatron, now engaged in Run II, has significant potential to discover a light Standard Model (SM) or Minimal Supersymmetric Standard Model (MSSM) Higgs boson, with mass up to about $M_H \lesssim 130$ GeV [1]. However, it will have very little capability to determine the overarching model that governs EWSB if a Higgs candidate is observed. The CERN Large Hadron Collider (LHC), on the other hand, will have considerably expanded capability to discover and measure almost all the quantum properties of a SM Higgs of any mass or several of the MSSM Higgs bosons over the entire MSSM parameter space [2–6]. While this is certainly very promising for future studies of EWSB, very little attention has been given recently to non-SM/MSSM theories of gauge boson and fermion mass generation.

Of particular concern to us are the more modern dynamical models of EWSB. While dynamical models have historically had many theoretical problems as well as conflicts with data, and broad classes have been ruled out, there are still viable models worthy of investigation in light of the capabilities of the current generation of experiments. We address here the theory of topcolor assisted technicolor (TC2) [7], specifically type I [8]. This model is still consistent with experiment [9]. We first outline the model in Sec. II, discuss the phenomenology of the model in Sec. III, and then present conclusions and the outlook for upcoming experiments. Details of some of the analytical calculations are presented in the Appendices.

II. THE TOPCOLOR ASSISTED TECHNICOLOR MODEL

Dynamical theories of fermion mass generation, the most viable of which is extended technicolor (ETC), typically have difficulty accommodating the large top quark mass. TC2 was proposed to assuage this problem, by having two separate strongly interacting sectors. One (topcolor, or TC) provides for the large top quark mass but has comparatively little contribution to EWSB, while the other (ETC) is responsible for the bulk of EWSB but contributes almost nothing to m_t . Details of TC2 may be found in Ref. [7]. Here, we briefly review the characteristics most relevant for discussion of its phenomenology.

Topcolor gauge interactions cause top quark pair condensation at some scale Λ via a strong four-fermion interaction

$$\frac{g^2}{\Lambda^2} \bar{\psi}_L t_R \bar{t}_R \psi_L, \quad (1)$$

where the fields are the SM third generation matter fields in $SU(2)$ doublet and singlet representation. The resulting chiral symmetry breaking yields a set of Goldstone bosons. The interaction of these bosons and the condensate may be written as an $SU(2)$ field Φ in exponential form:

$$\Phi_{TC} = e^{i\vec{\tau} \cdot \vec{\pi}/f_\pi} \begin{pmatrix} (f_\pi + H_{TC})/\sqrt{2} \\ 0 \end{pmatrix}, \quad (2)$$

where f_π is the vacuum expectation value (vev) of the top quark pair condensate, and $\vec{\tau}$ are the Pauli matrices. Note that the hypercharge of Φ is -1 . Type I topcolor contains an extra $U(1)$ which tilts the fermion interaction to disallow condensation of a $b\bar{b}$ condensate as well. Here, the gauge interactions are universal so flavor changing neutral currents are suppressed. A similar condensation $\langle \bar{T}_L T_R \rangle$ of technifermions occurs in the ETC sector, with its own vev v_T , and one may write the $SU(2)$ doublet Φ_{ETC} in the same form.

The Pagels-Stokar formula [10] gives the value of the vev f_π in terms of the number of topcolors, the top quark mass, and the scale at which the condensation occurs:

$$f_\pi^2 \simeq \frac{N_c}{16\pi^2} m_t^2 \left[\ln \left(\frac{\Lambda^2}{m_t^2} \right) + K \right], \quad (3)$$

where K is a constant of order 1. For condensation around the EWSB scale of 1 TeV, $f_\pi \simeq 60$ GeV, but it should be understood that this is only a rough guide, and f_π may in fact be somewhat lower or higher, say in the range 40 – 80 GeV. Allowing f_π to vary over this range does not qualitatively change our conclusions and has only minimal impact on our quantitative results. Therefore, we use the value $f_\pi = 60$ GeV throughout our analysis as a convenient baseline.

We linearize the theory and rearrange the pions in two orthogonal linear combinations to form the longitudinal degrees of freedom of the weak gauge bosons and a triplet of “top-pions”, $\Pi^{0,\pm}$, which become physical degrees of freedom. (See Appendix A for details.) The top-pions are analogous to the neutral CP-odd and charged Higgs scalars of a two-Higgs doublet model (2HDM), of which the MSSM Higgs sector is a subset. Contributions to the top quark mass can come from both sectors, but the model assumes that the dominant contribution is from TC. The top quark Yukawa term in the Lagrangian, ignoring mixing between the two Higgs modes, is written as

$$\begin{aligned}
\mathcal{L}_{Yuk,t} = & -\frac{1}{\sqrt{2}} \left(Y_t f_\pi + \epsilon_t v_T \right) \bar{t} t \\
& -\frac{1}{\sqrt{2}} \left(Y_t H_{TC} + \epsilon_t H_{ETC} \right) \bar{t} t \\
& -\frac{i}{v\sqrt{2}} \left(Y_t v_T - \epsilon_t f_\pi \right) \Pi^0 \bar{t} \gamma^5 t .
\end{aligned} \tag{4}$$

where Y_t is the TC Yukawa coupling, and ϵ_t is a small ETC contribution. Once f_π is fixed, v_T is uniquely determined by the EWSB requirement that $f_\pi^2 + v_T^2 = v^2 \simeq (246 \text{ GeV})^2$. For $f_\pi = 60 \text{ GeV}$, we must have $v_T = 239 \text{ GeV}$. The measured top mass then fixes Y_t to be of order 3-4 for small ϵ_t . The maximal value of Y_t is $Y_{t,max} = 4.1$ occurs when $\epsilon_t = 0$.

The two CP-even Higgs modes in this effective 2HDM, labeled H_{TC} and H_{ETC} , are known as the “top-Higgs” and the “techni-Higgs”, respectively. Their masses can be estimated in the Nambu–Jona–Lasinio (NJL) model in the large- N_c approximation. For the top-Higgs this is found to be on the order of $M_H \simeq 2m_t$; for the techni-Higgs it is much higher. However, there is no reason to expect the NJL model to be correct, it only serves as a rough guide; the masses of the top- and techni-Higgs modes may in fact be very light. The top-pions on the other hand have masses proportional to ϵ_t and the mass of the color octet of TC gauge bosons, M_B . In the fermion bubble approximation this is

$$M_\Pi^2 = \frac{N_c \epsilon_t m_t^2 M_B^2}{8\pi^2 f_\pi^2} . \tag{5}$$

If $M_B \sim 1 \text{ TeV}$, the theory loosely predicts top-pions to lie in the mass range of about 100-300 GeV.

While topcolor does not give mass to the bottom quark directly, it can generate a contribution via instanton effects. This contribution, $m_b^* \leq m_b$, is approximately,

$$m_b^* \approx \frac{3km_t}{8\pi^2} \sim 6.6 k \text{ GeV} . \tag{6}$$

To get a limit on k , we use a bottom quark pole mass of $m_b \approx 4.8 \text{ GeV}$, so that the entire b quark mass would come from contribution due to topcolor instantons for $k \sim 0.73$. Since Eq. 6 is only a rough estimate we will use $k = 0.8$ as the maximum possible value in our analysis. The remaining m_B contribution is assumed to come from ETC, via a Yukawa coupling ϵ_b . The Lagrangian terms for the ETC bottom Yukawa and instanton sectors are

$$\begin{aligned}
\mathcal{L}_{Yuk,b} = & -\left(m_b^* + \frac{\epsilon_b v_T}{\sqrt{2}} \right) \bar{b} b \\
& -\frac{i}{v\sqrt{2}} \left(\frac{\sqrt{2}m_b^*}{f_\pi} v_T - \epsilon_b f_\pi \right) \Pi^0 \bar{b} \gamma^5 b .
\end{aligned} \tag{7}$$

For fixed f_π , this coupling depends only on k (ϵ_b is related to k by m_b), and has a zero at $k = 0.043$. Such a small non-zero value seems extraordinarily fine-tuned so we do not consider it as a special case further. A more interesting special case is where ETC has flavor universal Yukawa couplings, *i.e.* $\epsilon_b = \epsilon_t$. This can occur only for very large values of the topcolor Yukawa coupling, $Y_t \gtrsim 4$ (recall for our fixed value of f_π , $Y_{t,max} \sim 4.1$). At the lower limit of this bound, $k = 0$ and there is no topcolor instanton-induced b quark mass.

III. PHENOMENOLOGY OF THE MODEL

One immediately can see from Eq. 4 that the couplings of both the top-Higgs mode and the top-pion to top quarks are enhanced by a factor of several ($Y_t^{TC}/Y_t^{SM} \simeq 3 - 5$ for the top-Higgs and $(Y_t^{TC}v_T - \epsilon_t f_\pi)/vY_t^{SM} \simeq 3 - 4$ for the top-pion) relative to the SM. As a result, these states have a greatly enhanced top quark loop-induced coupling to gluons. Inclusive production, $gg \rightarrow \Pi^0$, thus occurs at a much greater rate than in the SM. This latter feature has been addressed previously in the literature, briefly in Ref. [11] and in more detail for a type II topcolor model in Ref. [12], and we do not discuss it here. Furthermore, we will not discuss either the top-Higgs or the techni-Higgs in this paper, leaving them for future analysis [13]. Instead, we concentrate our investigation the neutral top-pion, which has not been examined very closely in previous studies.

As the Π^0 is a CP-odd state, it does not couple to weak bosons at tree level. This limits the production modes at a hadron collider, as well as the possible decay modes. We therefore focus on single-top- and $t\bar{t}$ -associated production, and compare the TC2 rates to corresponding rates in both the SM and allowed regions of the MSSM. We also confine our focus to the mass region $M_{\Pi^0} < 2m_t$. For masses above the top quark pair threshold, decays to top quarks dominate, resulting in a rather large four top quark cross section that may be experimentally observable, as discussed in Ref. [14].

All our calculations are performed with parton-level Monte Carlo using CTEQ4L parton distribution functions [15] and $\alpha_s(M_Z) = 0.1185$. Both the factorization and renormalization scales are chosen as $\mu_{f,r} = m_t + \frac{1}{2}M_{\Pi^0}$. Matrix elements were generated with Madgraph [16] by adding the TC2 scalar sector to its model tables. We do not consider running of the Yukawa couplings. Since the values are unknown, that analysis seems premature. We show only a few representative choices of the possible couplings, to characterize the model's general behavior. If the Π^0 is observed, it will then be important to study higher order effects.

Decays of the neutral top-pion

After calculating the top-pion couplings to SM particles, we evaluate the dominant partial widths of the top-pion, decays into $b\bar{b}$, gg , and for $M_{\Pi^0} > 2m_t$, top quark pairs. The results are shown in Fig. 1 for different values of k in the $b\bar{b}\Pi^0$ instanton-induced coupling. For $M_\phi \lesssim 150$ GeV, the top-pion has a significantly larger width than a SM Higgs, but for $M_\phi \gtrsim 150$ GeV the top-pion is approximately an order of magnitude narrower than a SM Higgs due to the lack of tree level decay modes to weak bosons. Since the top-pion width remains less than a GeV for all masses below the top pair threshold, the top-pion would appear experimentally as a narrow resonance, at the limit of detector width resolution in any decay channel. This is in contrast to the SM Higgs, where the width already exceeds detector resolution by about $M_H \approx 220$ GeV. The implication is that the total width of the top-pion can be determined only indirectly.

For $M_{\Pi^0} < 2m_t$, it is important to note that even if topcolor does not contribute a mass to the b quark (*i.e.* $k = 0$) there is still a small branching ratio to b quarks, although for $M_{\Pi^0} \gtrsim 150$ GeV this quickly becomes negligible. If instead $k = k_{max} \sim 0.8$, even at $M_{\Pi^0} = 100$ GeV the branching ratio to gluons is about 5%, the smallest it ever gets. For larger top-pion masses or more moderate values of k , there is typically a rather large branching ratio to gluons. We will later place rough limits on what we expect $\sigma \cdot \text{BR}$ to be for each decay mode as a function of Y_t and k .

For $M_{\Pi^0} > 2m_t$, the top-pion total width exceeds the SM Higgs total width by a factor 3-5, depending on the choice of Y_t . In this region, decays to top quark pairs dominate the width to such a degree that their branching ratio is effectively unity; all other decay modes may be ignored.

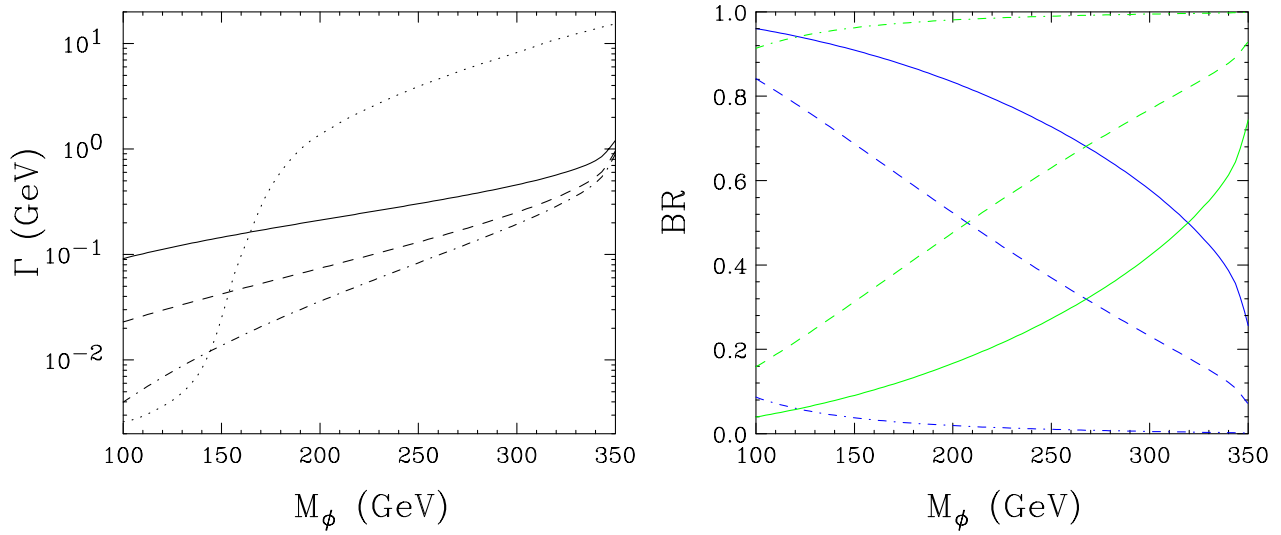


Figure 1. Total width (left) and dominant branching ratios (right) of the neutral top-pion, as a function of k for fixed $Y_t = 4.0$. Shown are the curves for k is 0.8 (solid), 0.4 (dashed) and 0 (dot-dashed). The SM Higgs total width is shown by the dotted line in the left panel. In the right panel, $BR(b\bar{b})$ are in blue (downward sloping), and $BR(gg)$ are in green (upward sloping).

Single top associated production

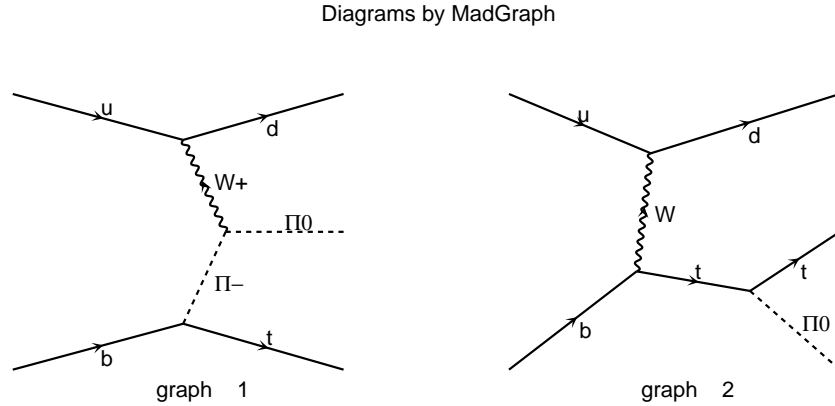


Figure 2. t -channel W single top associated Π^0 production. As in the MSSM there is a strong cancellation between the two diagrams, leading to small, almost certainly unobservable rates.

The largest single top production cross section at the Tevatron ($\sqrt{s} = 2.0$ TeV) is s-channel production, $u\bar{d} \rightarrow W^* \rightarrow t\bar{b}$. t-channel production, $ug \rightarrow dt\bar{b}$, dominates at the LHC ($\sqrt{s} = 14$ TeV). One may easily estimate that at either the Tevatron or LHC, even if s-channel single top associated production of a neutral top-pion is enhanced relative to the SM Higgs rate by $\sim 3^2$, an order of magnitude, this is not enough to be observed [17]. t-channel production is a different story. In this case there is a strong cancellation in the SM between the graphs where the Higgs is radiated off the t-channel W boson or off the final state top quark, which preserves unitarity at high energies [17]. Combined with the rather large background rates, this renders SM Higgs single top associated production unobservable at both the Tevatron and LHC. Even in the MSSM it is difficult to achieve a significant enough

enhancement to hope for much improved prospects. But in TC2, the neutral top-pion cannot be emitted from the t-channel W , so one would naïvely expect cancellations to be absent and the rate to be considerably larger. Unfortunately, there is a $W^+\Pi^-\Pi^0$ vertex, as shown in Fig. 2, which leads to a similar strong cancellation between the diagrams, again as required by unitarity (see Appendix B for details). Since at the LHC the top-pion production cross section is never more than a factor two larger than for the SM Higgs, we believe this channel is not useful and do not consider it further.

Top quark pair associated production

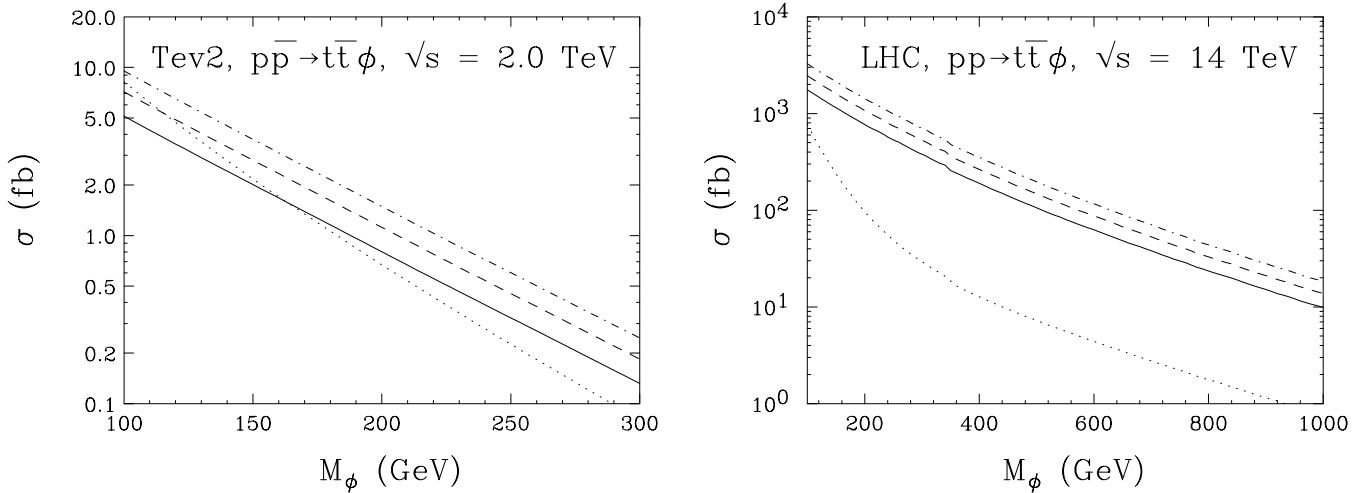


Figure 3. Total $t\bar{t}\Pi^0$ v. Standard Model $t\bar{t}H$ cross sections at the Tevatron (left) and LHC (right). TC2 model input is $f_\pi = 60$ GeV and $Y_t = 3.0$ (solid), 3.5 (dashed), and 4.0 (dotdashed). The SM cross sections are shown by the dotted curves.

The situation is very different for $t\bar{t}\Pi^0$ production as there are no cancellations between diagrams. The cross section at the Tevatron, shown in the left panel of Fig. 3, is comparable to that for SM $t\bar{t}H$ production for $M_{\Pi^0} \lesssim 150$ GeV, varying within a factor of several smaller to few larger. At larger Π^0 masses, $M_{\Pi^0} \gtrsim 150$ GeV, the TC rate is always larger, although the total rate is not enough to yield enough events [18]. That the rate is only comparable rather than significantly larger, as one would guess from the relative magnitude of the quark-quark-scalar couplings, is due to a different sort of cancellation: since the $t\bar{t}\Pi^0$ vertex contains a γ^5 , due to the CP-odd nature of the scalar, there is destructive interference between the $p_{in} \cdot p_{out}$ and m_t^2 terms in the Dirac structure of the amplitude. The Tevatron runs at a partonic center of mass energy where the terms are of comparable size, so the overall coupling enhancement of $\approx 3^2$ is unfortunately countered; if the γ_5 were not present, the cross section at the Tevatron would be larger by an order of magnitude [13].

Prospects for observation of $t\bar{t}H_{SM}$ events at the Tevatron were initially believed to be good for $M_H \lesssim 135$ GeV [18], but recent NLO calculations of $p\bar{p} \rightarrow t\bar{t}H_{SM}$ revealed an unexpected suppression rather than enhancement [19], which make the search much more difficult.¹ It is not yet known what

¹We note as an aside that $t\bar{t}A$ production in the MSSM is essentially never observable, as the cross section is always at least one order of magnitude smaller than the $t\bar{t}H$ rate of equal scalar mass [20].

the NLO result is for pseudoscalar production in association with top quark pairs at hadron colliders², so we cannot make definitive comments on the potential observability of this channel. The slightly lower cross sections for low Π^0 mass suggest that $t\bar{t}\Pi^0$ production is likely to be missed at the Tevatron, at least for small to moderate Y_t , but this should be viewed as a challenge to the machine and detector groups. Observing or ruling out TC2 based on its neutral pseudoscalar content will at the very least be extremely difficult at the Tevatron unless the machine performs exceedingly well.

A completely different paradigm will reign at the LHC. From a recent study with detector simulation [21], it is known that a SM Higgs of mass $M_H = 120$ GeV can be discovered in the $t\bar{t}H \rightarrow \ell\nu jjb\bar{b}b\bar{b}$ channel. That CMS study found that the backgrounds can be reduced to the level of the signal, $S/B \sim 1/1$, yielding a statistical significance of about 12σ . The study used the sample consisting of one top quark decaying hadronically and the other leptonically, $\approx 1/3$ of the total $t\bar{t}H$ event sample.

We predict that any $t\bar{t}\Pi^0; \Pi^0 \rightarrow b\bar{b}$ rate that is more than half the SM rate for the same M_ϕ will be observable at greater than 5σ . Examining the left panel of Fig. 4, for $M_{\Pi^0} = 120$ GeV this corresponds to $Y_t = 3.0$ and very small k , close to 0 (ignoring the exact zero at $k \approx 0.05$). For larger Y_t , the top-pion signal only becomes stronger, as the production cross section increases faster than $BR(b\bar{b})$ falls off. (This behavior holds generally for all Π^0 masses.) It is manifest that any region of parameter space with $\sigma \cdot BR(b\bar{b}) \gtrsim 300$ fb is likewise accessible. In fact the situation is much better, since the $t\bar{t}b\bar{b}$ background falls off very quickly with increasing $m_{b\bar{b}}$. However, we cannot match the level of sophistication presented in Ref. [21], and a parton-level Monte Carlo calculation would be a misleading comparison, so we leave the details of reach in this channel to future work by detector collaborations. We do note, however, that for the obviously very large region of parameter space where statistical significance would be $\gg 5\sigma$, the methods of Ref. [5] should also allow for confirmation of the pseudoscalar nature of the resonance.

For larger masses M_{Π^0} , it may be possible to observe the decay mode $\Pi^0 \rightarrow gg$ over some region of TC2 parameter space. We know of no other model where this is possible. To illustrate our claim we examine a few points in parameter space in Table I. Here we calculate the signal and QCD $t\bar{t} + jj$ backgrounds [22] at parton level with full matrix elements, including the decay $\Pi^0 \rightarrow gg$. We consider the final state where one top quark decays hadronically and the other decays leptonically, providing a hard lepton for triggering. We do not attempt to include detector effects, but we do include some major detector efficiencies such as b jet tagging (60% each) and lepton ID (85%), which reduces the captured rates considerably. We also apply the rather severe kinematic cuts needed to satisfy the experimental criteria for high luminosity running:

$$\begin{aligned} p_T(j) &> 30\text{GeV}, & |\eta(j)| &< 4.5, \\ p_T(b) &> 30\text{GeV}, & |\eta(b)| &< 2.5, \\ p_T(l) &> 15\text{GeV}, & |\eta(l)| &< 2.5, \\ \cancel{p}_T &> 50\text{GeV}, & \Delta R_{ij} &> 0.4. \end{aligned} \tag{8}$$

In addition, we require an additional cut $p_T > 40(50)$ GeV on the jets from decay of the Π^0 for $M_{\Pi^0} = 200(300)$ GeV. As the Π^0 is a narrow state even at the higher mass, we examine signal v.

²NLO results for $e^+e^- \rightarrow t\bar{t}H_{SM}$ turned out to be a poor guide for $p\bar{p} \rightarrow t\bar{t}H_{SM}$ at the Tevatron, so the known results for $e^+e^- \rightarrow t\bar{t}A_{MSSM}$ are also likely not so useful here.

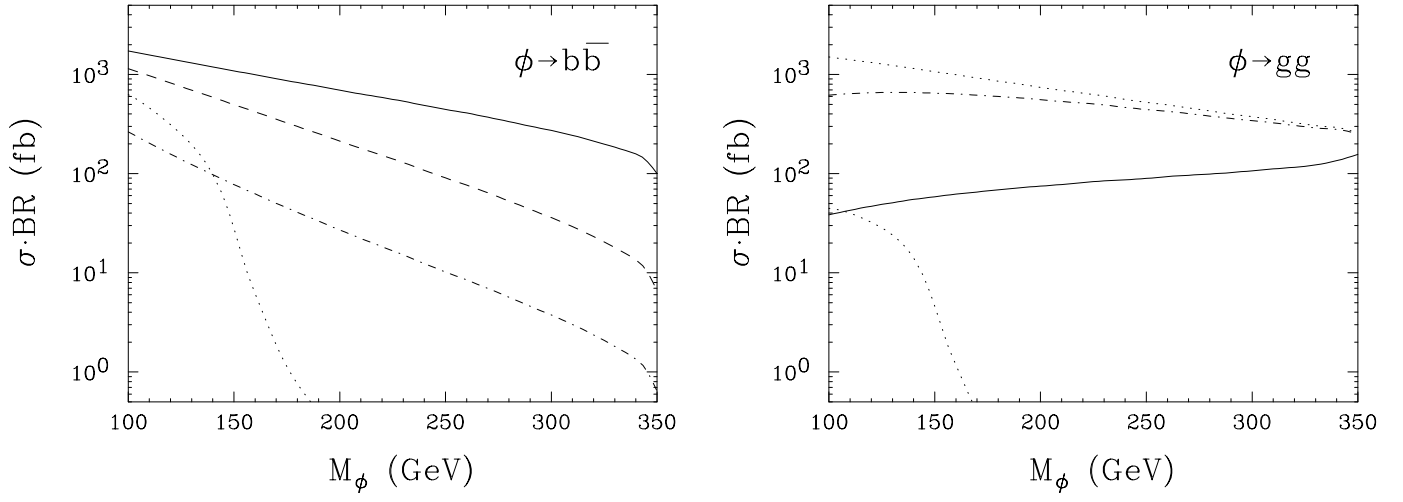


Figure 4. We show the top-pion production cross sections for $Y_t = 3.0$, multiplied by the branching ratios to $b\bar{b}$ (left) and gg (right), for various values of k : 0.8 (solid), 0.2 (dashed), 0 (dotdashed). The SM Higgs rates are shown by the dotted lines.

background in a ± 20 GeV bin around the central value. Due to the lack of detailed detector simulation, this comparison should be taken only as a rough guide for the reach available in this channel. Our goal is to show the potential distinctive characteristics of the TC2 model.

M_{Π^0} (GeV)	Y_t, k	σ_S (fb)	σ_B (fb)	N_S	N_B	S/B	$N_S/\sqrt{N_B}$
200	3.0, 0.8	2.2	680	140	62,400	1/440	0.6
200	3.0, 0.2	16.4	680	1050	62,400	1/60	4.2
200	4.0, 0.8	4.0	680	260	62,400	1/240	1.0
200	4.0, 0.2	19.9	680	1280	62,400	1/50	5.1
300	3.0, 0.8	3.3	290	210	26,600	1/130	1.3
300	3.0, 0.2	10.6	290	680	26,600	1/40	4.2
300	4.0, 0.8	9.2	290	590	26,600	1/45	3.6
300	4.0, 0.2	20.6	290	1320	26,600	1/20	8.1

TABLE I. Cross sections for the topcolor assisted technicolor signal $pp \rightarrow t\bar{t}\Pi^0 \rightarrow b\bar{b}l\nu jjgg$ (1 leptonic and 1 hadronic decay of the top quarks) and background $pp \rightarrow t\bar{t}jj \rightarrow b\bar{b}l\nu jjjj$ at the LHC, $\sqrt{s} = 14$ TeV. The number of events are calculated for 300 fb^{-1} of integrated luminosity, a universal efficiency factor of 0.31 for particle ID, and an efficiency factor of 0.7 for the mass bin capture of the signal only.

Table I reveals that the $\Pi^0 \rightarrow gg$ decay mode is probably observable only for large Y_t or very small k . While the number of background events is very large, S/B and total number of signal and background events are quite similar to the SM $gg \rightarrow H \rightarrow \gamma\gamma$ search at the LHC, which has been shown to be accessible [2]. Our estimate also makes no attempt to utilize the complex nature of these final states, which has elsewhere been shown to yield significant improvements beyond our simple approach [21]. The Table suggests that this mode may be able to provide discovery coverage over regions of parameter space where the $\Pi^0 \rightarrow b\bar{b}$ mode is not accessible.

IV. CONCLUSIONS

We have outlined the relevant features of topcolor assisted technicolor models of type I, which do not possess flavor-changing neutral currents. TC2 is effectively a two Higgs doublet model where one doublet is primarily responsible for giving mass to the top quark and the other is the dominant contributor to electroweak symmetry breaking. This results in strongly enhanced couplings of the neutral pion mode, Π^0 (analogous to the CP-odd scalar A in the MSSM), to top quarks. Direct observation of the Π^0 via its enhanced top quark couplings would be confirmation that the EWSB sector realized in nature is not SM or part of the MSSM.

The mass of the Π^0 is loosely expected to be fairly light, in the 100–300 GeV region. It would decay predominantly to $b\bar{b}$ or gg final states, depending on its mass and how much instanton contribution to the b quark mass comes from topcolor, which cannot be determined theoretically with much confidence and is simply parameterized.

Our investigation reveals that, as in other EWSB models involving one or two Higgs doublets, the single top associated production mode has a very small, almost certainly unobservable cross section at any hadron collider. $t\bar{t}$ associated production, on the other hand, benefits from a greatly enhanced cross section at the LHC (although not, alas, at the Tevatron). We find that $t\bar{t}\Pi^0; \Pi^0 \rightarrow b\bar{b}$ events should be easily discernible over much of the TC2 parameter space. In contrast, only very small, unobservable rates for $t\bar{t}A$ production (as well as $t\bar{t}h$ production for $M_h \gtrsim 140$ GeV) are predicted for the MSSM. Furthermore, for large Y_t or very small $b\bar{b}\Pi^0$ coupling, and large M_{Π^0} (but less than the top pair threshold), the decay $\Pi^0 \rightarrow gg$ is likely to be visible over the $t\bar{t} + jj$ background. We anticipate that this will provide for more complete coverage of TC2 parameter space, but deserves a detailed detector simulation to explore fully. For top-pion masses above the top pair threshold, the four top production cross section is greatly enhanced, although we do not address this signature.

If TC2 is the correct model describing nature, and the top-pion is observed at the LHC, there is still a long way to go toward determining the location of the model in parameter space. Ignoring the potential, there are effectively six unknowns ($f_\pi, v_T, Y_t, \epsilon_t, k, \epsilon_b$), but fewer constraints: the top and bottom quark masses, EWSB v , and the $t\bar{t}\Pi^0$ production cross section times the branching ratio to either b quarks or gluons. Our analysis shows that it is not very likely for the total rate to be determinable at a hadron collider. But by also observing another production mode in the same decay channel, such as $gg \rightarrow \Pi^0 \rightarrow b\bar{b}$, one can get around having to know either k or ϵ_b . While the number of unknowns is reduced to four, the number of measurements is still effectively three. This leaves the system underdetermined, so that additional measurements would be necessary, such as the rate of H_{TC} production times $BR(b\bar{b}, gg)$ rate in either gluon fusion or top quark associated production.

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APPENDIX A: THE TC2 LAGRANGIAN

We begin by writing the effective TC2 Lagrangian in linearized form. The kinetic term is

$$\mathcal{L}_{kin} = \left(D_\mu \Phi_{TC} \right)^\dagger \left(D^\mu \Phi_{TC} \right) + \left(D_\mu \Phi_{ETC} \right)^\dagger \left(D^\mu \Phi_{ETC} \right), \quad (\text{A1})$$

where the $SU(2)$ doublets Φ have the form

$$\Phi_{TC} = \begin{pmatrix} (f_\pi + H_{TC} + i\pi_{TC}^0)/\sqrt{2} \\ i\pi_{TC}^- \end{pmatrix}, \quad (\text{A2a})$$

$$\Phi_{ETC} = \begin{pmatrix} (v_T + H_{ETC} + i\pi_{ETC}^0)/\sqrt{2} \\ i\pi_{ETC}^- \end{pmatrix}, \quad (\text{A2b})$$

and the covariant derivative is

$$D_\mu = \partial_\mu + i\frac{g_Y}{2}YB_\mu + i\frac{g}{2}\tau_i W_\mu^i. \quad (\text{A3})$$

The hypercharge of the doublets is $Y = -1$, and g is g_{weak} . We make the following redefinition of fields:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \quad (\text{A4})$$

$$W_\mu^3 = Z_\mu \cos \theta + A_\mu \sin \theta, \quad (\text{A5})$$

$$B_\mu = -Z_\mu \sin \theta + A_\mu \cos \theta. \quad (\text{A6})$$

After replacement of the physical vector boson fields, the $D_\mu \Phi_i$ term for each doublet will be of the form

$$\begin{aligned} D_\mu \Phi_i = & \begin{pmatrix} \frac{1}{\sqrt{2}}(\partial_\mu H_i + i\partial_\mu \pi_i^0) \\ i\partial_\mu \pi_i^- \end{pmatrix} + \frac{ig_Z}{2} Z_\mu \begin{pmatrix} \frac{1}{\sqrt{2}}(v_i + H_i + i\pi_i^0) \\ -i(1 - 2\sin^2 \theta_W)\pi_i^- \end{pmatrix} \\ & + eA_\mu \begin{pmatrix} 0 \\ \pi_i^- \end{pmatrix} + \frac{ig}{2} \begin{pmatrix} i\sqrt{2}W_\mu^+ \pi_i^- \\ W_\mu^- (v_i + H_i + i\pi_i^0) \end{pmatrix}. \end{aligned} \quad (\text{A7})$$

where $g_Z = g/\cos \theta_W$ and $e = g \sin \theta_W$. After expanding the terms in Eq. A1, we form orthogonal linear combinations of the fields $\pi_i^{0,\pm}$,

$$w^{0,\pm} = \frac{f_\pi \pi_{TC}^{0,\pm} + v_T \pi_{ETC}^{0,\pm}}{v} \quad (\text{Goldstone bosons}), \quad (\text{A8})$$

$$\Pi^{0,\pm} = \frac{v_T \pi_{TC}^{0,\pm} - f_\pi \pi_{ETC}^{0,\pm}}{v} \quad (\text{physical top - pions}), \quad (\text{A9})$$

where $v^2 = f_\pi^2 + v_T^2 = (246 \text{ GeV})^2$.

After rearrangement the Feynman rules can simply be read off. At this point we reverse the flow of all bosons from incoming to outgoing, to match the treatment used in MADGRAPH/HELAS. The coefficient of each term is the HELAS coupling. Table II lists the 3-point gauge couplings for all physical fields; the Goldstone boson and 4-point couplings are not listed for brevity.

$Z^\mu Z_\mu H_{TC}$	$\frac{1}{2} f_\pi g_Z^2$	$Z^\mu Z_\mu H_{ETC}$	$\frac{1}{2} v_T g_Z^2$
$W^{+\mu} W_\mu^- H_{TC}$	$\frac{1}{2} f_\pi g^2$	$W^{+\mu} W_\mu^- H_{ETC}$	$\frac{1}{2} v_T g^2$
$Z^\mu H_{TC} \Pi^0$	$-\frac{i}{2} g_Z \frac{v_T}{v} (p_\mu^H - p_\mu^0)$	$Z^\mu H_{ETC} \Pi^0$	$+\frac{i}{2} g_Z \frac{f_\pi}{v} (p_\mu^H - p_\mu^0)$
$Z^\mu \Pi^- \Pi^+$	$g_Z (1 - 2 \sin^2 \theta_W) (p_\mu^- - p_\mu^+)$	$A^\mu \Pi^- \Pi^+$	$e (p_\mu^- - p_\mu^+)$
$W^{-\mu} H_{TC} \Pi^+$	$-\frac{i}{2} g \frac{v_T}{v} (p_\mu^H - p_\mu^+)$	$W^{+\mu} \Pi^- H_{TC}$	$+\frac{i}{2} g \frac{v_T}{v} (p_\mu^- - p_\mu^H)$
$W^{-\mu} H_{ETC} \Pi^+$	$+\frac{i}{2} g \frac{f_\pi}{v} (p_\mu^H - p_\mu^+)$	$W^{+\mu} \Pi^- H_{ETC}$	$-\frac{i}{2} g \frac{f_\pi}{v} (p_\mu^- - p_\mu^H)$
$W^{-\mu} \Pi^0 \Pi^+$	$-\frac{1}{2} g (p_\mu^0 - p_\mu^+)$	$W^{+\mu} \Pi^- \Pi^0$	$-\frac{1}{2} g (p_\mu^- - p_\mu^0)$

TABLE II. MADGRAPH/HELAS 3-point TC2 gauge couplings for the physical fields; Goldstone boson and 4-point couplings are not listed. All bosons (charge and momentum) flow out in the HELAS convention.

Using the same scalar $SU(2)$ doublets in Eq. A2, the Yukawa term in the Lagrangian is written as

$$\mathcal{L}_Y = -Y_t \left(\bar{\Psi}_L \Phi_{TC} t_R + \bar{t}_R \Phi_{TC}^\dagger \Psi_L \right) - \epsilon_t \left(\bar{\Psi}_L \Phi_{ETC} t_R + \bar{t}_R \Phi_{ETC}^\dagger \Psi_L \right), \quad (\text{A10})$$

where Ψ_L is the $SU(2)_L$ top-bottom quark doublet as usual. Rearrangement of the pion fields results in the Feynman rules for the quark Yukawa interactions with the top-Higgs, techni-Higgs and top-pions, shown in Table III.

$H_{TC} \bar{t}_R t_L$	$-\frac{1}{\sqrt{2}} Y_t$	$H_{TC} \bar{t}_L t_R$	$-\frac{1}{\sqrt{2}} Y_t$
$H_{ETC} \bar{t}_R t_L$	$-\frac{1}{\sqrt{2}} \epsilon_t$	$H_{ETC} \bar{t}_L t_R$	$-\frac{1}{\sqrt{2}} \epsilon_t$
$\Pi^0 \bar{t}_R t_L$	$+\frac{i}{v\sqrt{2}} (Y_t v_T - \epsilon_t f_\pi)$	$\Pi^0 \bar{t}_L t_R$	$-\frac{i}{v\sqrt{2}} (Y_t v_T - \epsilon_t f_\pi)$
$\Pi^- \bar{t}_R b_L$	$+\frac{i}{v} (Y_t v_T - \epsilon_t f_\pi)$	$\Pi^+ \bar{b}_L t_R$	$-\frac{i}{v} (Y_t v_T - \epsilon_t f_\pi)$

TABLE III. MADGRAPH/HELAS Yukawa quark-quark-scalar TC2 couplings. Y_t is the large topcolor top quark Yukawa, and ϵ_t is the ETC Yukawa giving a small contribution to the top quark mass. All bosons (charge and momentum) flow out in the HELAS convention.

APPENDIX B: SINGLE-TOP ASSOCIATED Π^0 PRODUCTION

To examine the analytical behavior of single top associated Π^0 production at hadron colliders we write the amplitudes for the two Feynman graphs in Fig. 2 in the effective- W approximation as in Refs. [17,23]:

$$\bar{u}_t \left(\frac{i}{2v} (Y_t v_T - \epsilon_t f_\pi) (1 - \gamma_5) \right) u_b \frac{-1}{(p_t - p_b)^2 - M_\Pi^2} \frac{g}{2} (p_\mu^+ - p_\mu^0) \epsilon^\mu, \quad (\text{B1})$$

$$\bar{u}_t \left(-\frac{i}{v\sqrt{2}} (Y_t v_T - \epsilon_t f_\pi) \gamma_5 \right) \frac{-(\not{p}_b + \not{k} + m_t)}{(p_b + k)^2 - m_t^2} \frac{g}{2\sqrt{2}} \gamma_\mu (1 - \gamma_5) u_b \epsilon^\mu. \quad (\text{B2})$$

The same couplings appear in both diagrams. Using the high energy limit $\epsilon^\mu = k^\mu/M_W + \mathcal{O}(M_W/k^0)$, the first term reduces completely to the couplings coefficient and a simple Dirac structure,

$$-\frac{ig}{4vM_W} (Y_t v_T - \epsilon_t f_\pi) \bar{u}_t (1 - \gamma_5) u_b = -C \bar{u}_t (1 - \gamma_5) u_b. \quad (\text{B3})$$

The second term reduces almost as neatly, in the quite reasonable approximation for high energy scattering that $m_b \sim 0$:

$$C \left[\bar{u}_t (1 - \gamma_5) u_b - \frac{m_t}{(p_b + k)^2 - m_t^2} \bar{u}_t \not{p}_\Pi (1 - \gamma_5) u_b \right]. \quad (\text{B4})$$

The first term of Eq. B4 cancels the contribution from the first graph in Eq. B3, leaving a term that satisfies the unitarity constraint at high energy [17,23].

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